Boundary condition for diffusion at a thin membrane determined from experimental data

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We present a method of deriving a boundary condition for diffusion at a thin membrane from experimental data. Within this method the Laplace transform of a boundary condition is assumed to be in the form

$$\hat{C}_2(0^+,p) = \hat{\Phi}(p)\hat{C}_1(0^-,p) \;, \tag{1}$$

where $\hat{\Phi}(p)$ is a function to be determined. Next, we find the Laplace transform of some theoretical function containing Φ, which is a relatively easy to measure experimentally. Then, this function is also determined by means of a numerical calculation of the Laplace transform of the experimental data obtained for normal diffusion of ethanol in water in a system with a nephrophan membrane. Finally, comparing both Laplace transforms mentioned above, we find the function Φ. The derived boundary condition at a membrane contains a term with a Riemann-Liouville fractional time derivative

$$\alpha C_2(0^+,t) + \beta \frac{\partial^{1/2}}{\partial t^{1/2}} C_2(0^+,t) = C_1(0^-,t) \;. \tag{2}$$

Such a form of the boundary condition shows that particles transfer through a thin membrane is a "long-memory process." The presented method is an example that an important part of the mathematical model of physical processes may be derived directly from experimental data.

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